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# Spherical Gravitational Collapse and Accretion - Exact General Relativistic Description

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In this paper, we consider the problems of spherical gravitational collapse and accretion using a spherically symmetric, spatially homothetic spacetime, that is, as an exact solution [7] of the field equations of general relativity. Properties of matter like its equation of state determine whether the collapse becomes unstoppable or not since the spacetime under consideration admits any equation of state for matter in it. We can therefore describe the formation of a semi-stable object here. **A black hole may form in the unstoppable gravitational collapse and/or accretion but only as an infinite red-shift surface that is, however, not a null hyper-surface.** Therefore, spherical, astrophysical black holes will always be of this type. This result will have important implications for observational astrophysics and other considerations in General Relativity based on the conception of black hole as a null hyper-surface.

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## I. INTRODUCTION

A *physically realistic* gravitational collapse problem imagines matter, with regular initial data, collapsing under its self-gravity. The resultant compression of matter causes pressure to build-up in it. Further, matter compression generates heat and radiation because of either the onset of thermonuclear fusion reactions or other reasons. The radiation or heat then propagates through the space. The collapsing matter could stabilize to some size when its equation of state is such as to provide pressure support against gravity. If self-gravity dominates, the collapse continues to a spacetime singularity. The issue of Cosmic Censorship Hypothesis [1] relates to whether the singularity is visible to any observer or not, ie, whether it is naked or not.

Irrespective of whether a black hole or a matter condensate forms in the collapse, matter in the surroundings will accrete onto the central object. The accreting matter may, initially, be dust in the far away regions. However, it gets compressed as it moves closer to the central object and pressure must build up in it. In many such situations, heat and radiation partly escape the system and partly fall onto the central object together with the accreting matter.

Then, various stages of gravitational collapse can be distinguished on the basis of the properties of matter such as its equation of state that are different in different such stages. Therefore, any complete description of the collapse and accretion processes requires us to properly match spacetimes of various such stages to produce the final spacetime description. Note that the final spacetime will have to be a solution of the Einstein field equations. (Note that the equation of state at extremely high densities is not known.)

In any case, the final spacetime description of the gravitational collapse and/or the accretion process must admit a changing equation of state for collapsing/accreting matter. Further, such a spacetime must also admit an energy or heat flux during late collapse or accretion stages. To accomplish this process of matching different such spacetimes is a herculean, if not impossible, task.

Hence, another approach to this problem is essential. We could then demand that a spacetime describing the collapse and/or the process of accretion in its totality admits *any* equation of state and appropriate energy-momentum fluxes. In other words, the spacetime geometry should be obtainable from considerations that do not involve the equation of state for the matter in the spacetime. Furthermore, these considerations should result in a spacetime admitting energy-momentum fluxes. We now turn to precisely such considerations in General Relativity.

### Spatially Homothetic Spacetimes

The phenomenon of gravitation does not provide any length-scale or mass-scale for spatial distribution of matter properties. Note that the scale-independence of Newtonian gravity applies only to space and not to time. This is one of the fundamental, observational properties of gravitation.

Therefore, General Relativity as a theory of gravitation must not provide any length-scale for matter distributions. This is essentially the spatial scale-invariance property of the spacetime.

The scale-independence of gravity means that we can construct a gravitating object of any size and of any mass. Further, matter within such an object can be distributed in any desirable manner since gravity does not provide for the spatial distribution of matter within any gravitating object. (It is a separate question as to whether every such object will be stable or not.)

There must therefore exist in General Relativity a spacetime that allows matter density to be an arbitrary function of *each* of the three spatial coordinates. We emphasize that such a spacetime metric and all other metric forms that are reducible to it under non-singular coordinate transformations are the only solutions of the field equations of General Relativity that are consistent with gravity not possessing a length-scale for matter properties.

All other spacetimes that are *not* reducible to the aforementioned spacetime by non-singular coordinate transformations then *violate* this basic property of gravity that it has no length-scale for matter properties. Hence, such spacetimes must possess a specific length-scale or mass-scale for matter properties. In short, not every solution of the Einstein field equations respects the principle, namely, its spatial scale-independence.

The field equations of General Relativity are based on Einstein's equivalence principle which is, primarily, the principle of equality of the *inertial* and *gravitational* masses. It is the equivalence principle that leads to geometrization of gravity and, hence, from a variational principle, to the field equations of General Relativity. However, General Relativity does not automatically incorporate other basic properties of gravity, if any, for example that it does not specify any length-scale for matter properties. Therefore, we need to *separately* enforce the principle of "no length-scale for matter properties" on the solutions of the field equations of General Relativity.

In general, a homothetic Killing vector captures [2] the notion of the scale-invariance. The principle that gravity does not specify any length-scale for matter properties then requires the spacetime to admit, in general, three independent spacelike Homothetic Killing vectors corresponding to the three spatial dimensions. A spacetime that conforms to the spatial scale-invariance, to be called a *spatially homothetic spacetime*, is then required to admit, corresponding to each spatial coordinate, an appropriate *spatial* homothetic Killing vector $\mathbf{X}$ satisfying

$$\mathcal{L}_{\mathbf{X}} g_{ab} \;=\; 2\,\Phi\, g_{ab} \tag{1}$$

where $\Phi$ is an arbitrary constant. We then expect spatially homothetic spacetimes to possess arbitrary spatial characteristics for matter. This is also the broadest (Lie) sense of the scale-invariance of the spacetime leading not only to the reduction of the Einstein field equations as partial differential equations to ordinary differential equations but leading also to their separation.

There is another importance of the spatially homothetic spacetimes. We note that the field equations of General Relativity were arrived at by demanding only that these reduce to the Newton-Poisson equation in the weak gravity limit [3, 4]. But, the *field equations of any theory of gravity should contain the entire weak gravity physics due to the applicability of the laws of weak gravity to any form of matter displaying any physical phenomena.* The field equations are expected to be only the *formal* equality of the appropriate tensor from the geometry and the energy-momentum tensor of matter. Then, the field equations of General Relativity could have been obtained by imposing the requirement that these reduce to the single "equation of the entire weak gravity physics".

However, there is no "single" equation for the "entire weak gravity physics" since we include different physical effects in an ad-hoc manner in the newtonian physics.

But, *there can be a "single" spacetime containing the entire weak gravity physics.* Therefore, we need a principle to identify such a solution of the field equations. In the weak field limit, the spatial scale-invariance is the freedom of specification of matter properties through three independent functions of the three spatial coordinates, in general.

*The spatial scale-invariance is then the principle that could help us identify spacetimes containing the entire weak gravity physics.* Indeed. the spatial scale invariance identifies a single such spacetime [5]. It has appropriate energy-momentum fluxes, applicability to any form of matter and, hence, it contains the entire weak gravity physics. The newtonian law of gravitation then gets replaced by the single general relativistic spacetime [5] that contains all of the weak gravity physics. But, spatial scale-independence needs to be separately imposed on the field equations to obtain it.

Spatially homothetic spacetimes do not yield a naked singularity for initially non-singular, regular, spatial data for matter fields [5]. Such spacetimes always produce a black hole in the unstoppable gravitational collapse of spatially non-singular matter of any spatial properties. Due to

the freedom of the specification of spatial properties of matter in it, a spatially homothetic spacetime can describe the formation of some gravitating object, may that be a black hole, and accretion onto it in its entirety.

A general spatially homothetic spacetime, although known [5], is a complicated spacetime. The general problems of the formation of a gravitating object and accretion onto it are, therefore, highly involved problems to analyze in details. This is understandable since, in general, a collapsing newtonian object of arbitrary spatial characteristics will exhibit energy-momentum fluxes along all the cartesian coordinate directions. We will therefore have to deal with this complexity in all its generality. It is instructive, however, to begin with the simpler situation of spherical symmetry to gain insight into the physical nature of the gravitational collapse problem.

To fix ideas, we therefore begin in this paper with the simplest of such problems - a spherically symmetric problem - that considers the formation of a spherical gravitating object and accretion of matter onto it. To this end, we first recall from [7] the spherically symmetric, spatially homothetic spacetime and its properties. It will be seen that the temporal metric functions are determined by the properties of matter in it.

## II. SPACETIME OF ACCRETING, NON-ROTATING, SPHERICAL OBJECT

A general spherically symmetric spacetime admits a metric of the form

$$ds^2 = -\,A^2(r,t)\,dt^2 \;+\; B^2(r,t)\,dr^2 \;+\; C^2(r,t)\,\left[\,d\theta^2 \,+\, \sin^2\theta\,d\phi^2\,\right] \tag{2}$$

A spherically symmetric spacetime has only one spatial scale associated with it - the radial distance scale. The scale-independence of gravity then means that a spherically symmetric spacetime allows *arbitrary* radial properties for matter. The corresponding spatial homothetic Killing vector must then possess only a radial component in appropriate coordinates. Therefore, we impose [6] a *spatial* homothetic Killing vector

$$(\,0,\; f(r,t),\; 0,\; 0\,) \tag{3}$$

on the general spherically symmetric metric (2). This *uniquely* determines the spherically symmetric metric to that obtained in [7], namely

$$ds^2 = -\,y^2(r)\,dt^2 + \gamma^2\,(y')^2\,B^2(t)\,dr^2 + y^2(r)\,Y^2(t)\,\left[d\theta^2 + \sin^2\theta d\phi^2\right] \tag{4}$$

with $f(r,t) = y/(\gamma y')$, a prime indicating a derivative with respect to $r$ and $\gamma$ being a constant. (We shall always absorb the temporal function in $g_{tt}$ by suitable redefinition of the time coordinate.)

The coordinates are co-moving with the geometry. We can therefore define

$$D_t \;\equiv\; U^a\,\frac{\partial}{\partial x^a} \;=\; \frac{1}{y}\,\frac{\partial}{\partial t} \tag{5}$$

where $U^a$ is the four-velocity of the co-moving observer. Further, differentiation along an outward radial unit vector orthogonal to $U^a$ is given by

$$D_r \;\equiv\; \frac{1}{\gamma(y')B}\,\frac{\partial}{\partial r} \tag{6}$$

Then, the velocity of the fluid with respect to the co-moving observer is

$$V_r \;=\; D_t\,(yY) \;=\; \dot{Y} \tag{7}$$

The metric function $B(t)$ determines or gets determined by the energy flux in the spacetime. These characteristics of the temporal metric functions are important to the analysis of the physics of the gravitational collapse implied by (4).

The Einstein tensor for (4) has the following components in the chosen coordinate frame

$$G_{tt} = \frac{1}{Y^2} - \frac{1}{\gamma^2 B^2} + \frac{\dot{Y}^2}{Y^2} + 2\frac{\dot{B}\dot{Y}}{BY} \tag{8}$$

$$G_{rr} = \gamma^2 B^2 \left(\frac{y'}{y}\right)^2 \left[-2\frac{\ddot{Y}}{Y} - \frac{\dot{Y}^2}{Y} + \frac{3}{\gamma^2 B^2} - \frac{1}{Y^2}\right] \tag{9}$$

$$G_{\theta\theta} = -Y\ddot{Y} - Y^2\frac{\ddot{B}}{B} - Y\frac{\dot{Y}\dot{B}}{B} + \frac{Y^2}{\gamma^2 B^2} \tag{10}$$

$$G_{\phi\phi} = \sin^2\theta\, G_{\theta\theta} \tag{11}$$

$$G_{tr} = 2\frac{\dot{B}y'}{By} \tag{12}$$

where an overhead dot denotes a time derivative.

Notice that the $t$–$r$ component of the Einstein tensor is non-vanishing. Hence, the matter in the spacetime could be *imperfect* or *anisotropic* indicating that the energy-momentum tensor could be either of the following

$$^{\mathrm{I}}T_{ab} = (p + \rho)\,U_a U_b + p\,g_{ab} + q_a U_b + q_b U_a - 2\,\eta\,\sigma_{ab} \tag{13}$$

$$^{\mathrm{A}}T_{ab} = \rho\,U_a U_b + p_{||}\,n_a n_b + p_\perp P_{ab} \tag{14}$$

where $U^a$ is the matter four-velocity, $q^a$ is the heat-flux four-vector relative to $U^a$, $\eta$ is the shear-viscosity coefficient, $\sigma_{ab}$ is the shear tensor, $n^a$ is a unit spacelike four-vector orthogonal to $U^a$, $P_{ab}$ is the projection tensor onto the two-plane orthogonal to $U^a$ and $n^a$, $p_{||}$ denotes pressure parallel to and $p_\perp$ denotes pressure perpendicular to $n^a$. Also, $p$ is the isotropic pressure and $\rho$ is the energy density.

For the observer co-moving with geometry with four-velocity

$$U^a = \frac{1}{y}\,\delta^a{}_t \tag{15}$$

the kinematical quantities for the line element (4) are given by

$$\dot{U}_a = U_{a;b}U^b = \left(0,\, \frac{y'}{y},\, 0,\, 0\right) \tag{16}$$

$$\Theta_M = \frac{1}{y}\left(\frac{\dot{B}}{B} + 2\,\frac{\dot{Y}}{Y}\right) \tag{17}$$

$$\sigma \equiv \sigma^3{}_3 = \sigma^2{}_2 = -\frac{1}{2}\sigma^1{}_1 = \frac{1}{3\,y\,(2\eta)}\left(\frac{\dot{Y}}{Y} - \frac{\dot{B}}{B}\right) \tag{18}$$

where $\dot{U}_a$ denotes the four-acceleration of matter, $\Theta_M$ represents expansion of matter. Note that the shear tensor is trace-free and $\sigma$ represents the shear-scalar that is given by $\sqrt{6}\,\sigma$.

For $\Theta_M > 0$, the spacetime under consideration is expanding and, for $\Theta_M < 0$, the spacetime is contracting.

Now, the Einstein field equations with imperfect matter yield for (4)

$$\rho = \frac{1}{y^2}\left(\frac{\dot{Y}^2}{Y^2} + 2\frac{\dot{B}}{B}\frac{\dot{Y}}{Y} + \frac{1}{Y^2} - \frac{1}{\gamma^2 B^2}\right) \tag{19}$$

$$2\,\frac{\ddot{Y}}{Y} + \frac{\ddot{B}}{B} = \frac{2}{\gamma^2 B^2} - \frac{y^2}{2}\,(\rho + 3\,p) \tag{20}$$

$$3\,(2\,\eta)\,\sigma \;=\; \frac{1}{y^2}\left(\frac{\ddot{B}}{B} \;-\; \frac{\ddot{Y}}{Y} \;+\; \frac{\dot{B}\dot{Y}}{BY} \;-\; \frac{\dot{Y}^2}{Y^2} \;+\; \frac{2}{\gamma^2 B^2} \;-\; \frac{1}{Y^2}\right) \tag{21}$$

$$q \;=\; -\frac{2\dot{B}}{y^2\gamma^2 y' B^3} \tag{22}$$

where $q^a = (0, q, 0, 0)$ is the radial heat-flux vector.

Clearly, the radial function $y(r)$ is not determined by the field equations. Therefore, radial attributes of matter are *arbitrary*, meaning, unspecified, for the metric (4). This is in the manner of concentric spheres with each sphere allowed to possess any value of density, for example. This is the maximal freedom compatible with the assumption of spherical symmetry, we may note.

It also follows that the temporal metric functions $B(t)$ and $Y(t)$ get determined by the properties of matter such as an equation of state.

The point $r = 0$ will possess a locally flat neighborhood when $y'|_{r\sim 0} \approx 1/\gamma$. This condition must be imposed on any $y(r)$. Apart from this condition, the function $y(r)$ is arbitrary. Other physical considerations, such as those arising from the equation of heat transfer in the spacetime, could constrain the function $y(r)$.

The density is, for $y' > 0$, a decreasing function of $r$ corresponding to a region over-dense at its center. Therefore, for our purposes here, we will assume that there is only one over-dense region and, hence, $y' > 0$ throughout the spacetime. Then, we have that there is a "single" collapsing and accreting spherically symmetric object.

The spatial or radial nature of the heat flux is determined primarily by the sign of the quantity $-\dot{B}/y'$. The heat flux is positive, that is, heat flows from lower values of $r$ to higher values of $r$, when $y'$ and $\dot{B}$ have opposite signs. Then, with $y' > 0$, we require $\dot{B} < 0$ for the whole of the spacetime. Heat then flows from smaller values of $r$ to larger values of $r$. That is to say, heat flows from the central over-dense region to under-dense region surrounding it. This is then the general gravitational model with radially outward heat flux. A specific model is obtained for a specific choice of the radial function $y(r)$ with above conditions.

We remind the reader that additional conditions arising from the considerations of the stability of the stellar object etc. will constrain the radial metric function $y(r)$ in a manner similar to those obtainable for the Newtonian model of a star.

In general, we may define the mass function by

$$m(r,t) \;=\; \frac{yY}{2}\left(1 - \frac{Y^2}{\gamma^2 B^2} + \dot{Y}^2\right) \tag{23}$$

The field equations then imply

$$\frac{\partial m}{\partial r} \;=\; 4\pi\,\rho\,y^2 Y^3\,y' \tag{24}$$

$$\frac{\partial m}{\partial t} \;=\; -\,4\pi\,p\,y^3 Y^2\,\dot{Y} \tag{25}$$

For positive pressure $p$ in the spacetime and for $\dot{Y} < 0$, the mass accretes to the center and it is increasing in time in a collapsing situation. We may also define the luminosity of the central star as seen by a co-moving observer at location $r$ by

$$\mathcal{L} \;=\; 4\,\pi\,y^2\,Y^2\,q \tag{26}$$

**Semi-stable, radiating object**

Recall that $\dot{Y}$ is the radial velocity of matter with respect to the observer co-moving with the geometry. It can be positive for out-flowing matter, negative for in-flowing matter and zero for stable matter. Also, the mass accretion rate (25) will vanish for $\dot{Y} = 0$. Then, the temporal dependence of the mass function in (23) corresponding to $\dot{Y} = 0$ is purely due to the conversion of mass to radiation or heat. In this case, we also obtain

$$(2\eta)\,6\,\sigma \;=\; \gamma^2 B^2\,y\,y'\,q \tag{27}$$

for $\dot{Y} = 0$.

This is the "semi-stable" spherically symmetric, radiating object. The expressions for its density etc. follow from (19) - (22). In particular, we can write

$$\rho \;=\; \frac{1}{y^2}\left(1 \;-\; \frac{(y')y^2 B}{2|\dot{B}|}\,q\right) \tag{28}$$

$$\frac{\ddot{B}}{B} \;=\; \frac{2}{\gamma^2 B^2} - \frac{y^2}{2}\,(\rho + 3\,p) \tag{29}$$

by noting that the positivity of shear and heat flux, both, requires $\dot{B} < 0$ for $y' > 0$.

Note that the stability of such an object is not for all of the co-moving time. The properties of matter determine whether the object remains stable in this manner or not. Considerations such as those leading to the Chandrasekhar or the Oppenheimer-Volkov limits [8] are then possible. However, these require more details of the matter properties than are considered here.

### No collapse without heat generation

We may note that the heat generation at some stage during the gravitational collapse is expected on the basis of very general physical considerations of thermodynamic origin. Therefore, we must not obtain the situation of gravitational collapse without heat generation when we use the metric (4).

This is easily seen by taking $\dot{B} = 0$ so that the heat flux vanishes in the spacetime for all co-moving time since there is no heat generation. But, (18) implies that $\dot{Y} > 0$ for the shear-scalar to be positive. Then, (17) implies that matter in the spacetime of (4) is expanding and not contracting.

Consequently, we do not obtain the situation of gravitational collapse in the absence of heat generation in the spacetime of (4).

### Shell-crossing and Shell-focussing singularities

The singularities at locations for which $y' = 0$ are of shell-crossing type. These are however weak singularities since the curvature invariants do not blow up at locations for which $y' = 0$ [7]. Such locations have physical meaning in terms of the heat flow caustics.

The genuine spacetime singularities of the strong curvature, shell-focussing type exist when either $y(r) = 0$ for some $r$ or when the temporal functions vanish for some $t$. The possibility of $y(r) = 0$ for some $r$, however, means that we already have a spacetime singularity at that $r$. Therefore, we have, for spherically symmetric, spatially homothetic spacetimes, that $y(r) \neq 0$ at all $r$ for the non-singular initial data for matter fields.

(In general, for spatially homothetic spacetimes, the non-singularity of initial data for matter fields will be seen [5] to require the non-vanishing of the corresponding arbitrary functions of the spatial coordinates.)

Further, we note that the "physical" radial distance corresponding to the "coordinate" radial distance $\delta r$ is

$$\ell \;=\; \gamma(y')B\delta r \tag{30}$$

Then, a collapsing shell of matter forms the spacetime singularity in the present spacetime when $B(t) = 0$ is reached for it at some $t = t_s$. The temporal function $Y(t)$ determines the shear and the expansion in the spacetime together with the temporal function $B(t)$.

### Light Trapping Surface

A radially outgoing null vector of (4) is

$$\ell^a \partial_a \;=\frac{1}{y}\frac{\partial}{\partial t} \;+\; \frac{1}{\gamma y' B}\frac{\partial}{\partial r} \;=\; D_t + D_r \tag{31}$$

Light gets trapped inside a particular radial coordinate $r$ when the expansion of the above principle null vector vanishes. The formation of the outermost *light-trapping surface* (LTS) during any unstoppable collapse is then obtained by setting the expansion of (31) to zero.

The zero-expansion of (31) yields a condition only on the temporal metric functions as

$$y\,\Theta_M \;\equiv\; \frac{\dot{B}}{B} \;+\; 2\,\frac{\dot{Y}}{Y} \;=\; -\frac{3}{\gamma B} \tag{32}$$

When this condition is reached during the gravitational collapse light and, with it, matter trapping occurs. Note however that we are dealing here with the picture of the co-moving observer. For the co-moving observer $\dot{Y}$ is the radial velocity of matter and $\dot{B}$ determines the heat flux.

### No null hyper-surface or horizon

Note that (32) is not the condition for the formation of a null hyper-surface or the horizon in the spacetime. The formation of horizon or a null hyper-surface requires the normal of a hyper-surface to become null.

The existence of a spherical null hyper-surface requires that the norm of its normal vanishes at some $r$. If $n_a = (0, 1, 0, 0)$ is the normal to $r =$ constant hyper-surface, then we have $n^a n_a = 1/(\gamma^2 (y')^2 B^2)$ which, obviously, cannot vanish except for $y' \to \infty$ for some $r$ - an evidently degenerate-metric situation.

Moreover, we may, in terms of the mass function of (23), write

$$g_{rr} \;=\; \frac{Y^2\,(y')^2}{1 \,+\, V_r^2 \,-\, (2m/yY)} \tag{33}$$

Then, $g_{rr} \to \infty$ implies $Y(t) \to 0$. Importantly, therefore, there cannot form a spherical null hyper-surface at any radial location in (4).

Also, the coordinate speed of light in the spacetime of (4) is

$$\frac{dr}{dt} \;=\; \pm\,\frac{y}{\gamma(y')B} \tag{34}$$

This speed cannot vanish for any $r$ except in the case of an initially singular density distribution at

$r$ which we do not consider to be any serious, astrophysically meaningful, initial condition here.

Therefore, a null hyper-surface or horizon does not form in (4) for any non-singular initial data for matter fields.

### But, "Black Hole" forms

However, the light-trapping properties of gravity exist in the sense that gravity becomes strong enough to trap light in a spacetime region when the condition (32) is satisfied. (See also next section.)

We recall again that $\dot{Y}$ is the velocity of the fluid relative to the co-moving observer and that the temporal function $B(t)$ determines or is determined by the heat generation in the spacetime. The properties of matter then determine the temporal metric functions in the spacetime of (4). Therefore, depending on the properties of matter in the spacetime, trapping of light and matter occurs for (4).

The above is understandable as follows. A spherical star begins to collapse and the velocity of its matter increases as its collapse continues in the frame of the co-moving observer. It is only at some "instant" of the co-moving time that the curvature becomes strong enough to trap light and matter. The condition (32) determines this instant of the co-moving time.

This is the formation of a black hole at an instant of the co-moving time at which the light and, with it, matter get trapped in a strong gravitational field. This is the conception of a black hole that applies here and not that of a null hyper-surface, we may then note.

## III. SPHERICAL COLLAPSE, ACCRETION AND BLACK HOLE FORMATION

In the usual analysis of accretion onto a gravitating object, we generally consider a "central" object (that has already formed) and the surrounding matter (which is accreting onto it). However, this picture gets replaced by the problem of the formation of the central object and continued pile up of matter to the central region when we use the spatially homothetic spacetimes. This is primarily because these spacetimes have no spatial length scale and hence contain matter everywhere.

With the above feature of the spatially homothetic spacetimes in mind, we now turn to the problem of the continued accretion of matter onto a central object in the spherical collapse.

An observer co-moving with the geometry evaluates "various" quantities and interprets the observations of the star "as an asymptotic observer" for large $r$. On the other hand, an observer in the rest frame of the accreting matter is also important to the physical analysis of the problem under consideration. We therefore analyze the problem at hand by considering the observer in the rest frame of matter.

### In the rest frame of matter

The four-velocity of the matter fluid with respect to the co-moving observer is:

$$U^a \;=\; \left( U^t,\, U^r,\, 0,\, 0 \right) \tag{35}$$

Defining then the radial velocity of matter with respect to the co-moving observer as

$$V_r \;\equiv\; U^r / U^t \tag{36}$$

we then obtain from the metric (4):

$$U^a \;=\; \frac{1}{y\,\sqrt{\Delta}}\,(\,1,\, V_r,\, 0,\, 0\,) \tag{37}$$

$$\Delta \;=\; 1 \;-\; \gamma^2 \left( \frac{y'}{y} \right)^2 B^2\, V_r^2 \tag{38}$$

Now, if $d\tau_{CM}$ is a small time duration for the co-moving observer and if $d\tau_{RF}$ is the corresponding time duration for the observer in the rest frame of matter, then we have

$$d\tau_{CM} \;=\; \frac{d\tau_{RF}}{\sqrt{\Delta}} \tag{39}$$

Therefore, the co-moving observer waits for an infinite period of its time to receive a signal from the rest-frame observer when $\Delta \;=\; 0$. Equation (39) is also the red-shift formula. Clearly, therefore, $\Delta \;=\; 0$ is the *infinite red-shift surface.*

But, the above infinite red-shift surface is *not* a one-way membrane or a null hyper-surface since $U^a\, U_a \;=\; -1$ always. *This is an important point of distinction for the spatially homothetic spacetime of (4) from the spacetimes violating the spatial scale-invariance of gravity eg, the Schwarzschild spacetime for which a null hyper-surface exists and the infinite red-shift surface is also a null hyper-surface.*

Of course, the infinite red-shift surface separates the spacetime of (4) into two regions - one that can communicate to the far away zone and the black hole region that cannot. The inside and outside of the infinite red-shift surface are then causally disconnected regions of the spacetime of (4).

We have then the following possibilities

$$(\Delta > 0) \qquad |\gamma\,(y')\,B\,V_r| < y \qquad (40)$$
$$(\Delta = 0) \qquad |\gamma\,(y')\,B\,V_r| = y \qquad (41)$$
$$(\Delta < 0) \qquad |\gamma\,(y')\,B\,V_r| > y \qquad (42)$$

Matter with an initial density distribution determined by $y(r)$ begins to collapse under the condition (40) with initial velocity $V_{r,ini}$ and initial heat flux, determined by $B$, being small. The in-fall velocity of matter and heat flux grow as matter collapse progresses. Matter properties decide whether the collapse becomes unstoppable or not. Then, regions satisfying (41) and (42) form when condition (32) is reached.

In the case of accretion process, matter with inwardly directed radial velocity $V_r = -|\dot{Y}|$ and $\Delta > 0$ accretes onto the central object, may that be any, under suitable conditions such as the radiation pressure not reversing its radial velocity etc. (This is how the properties of matter play an important role in the collapse or accretion processes.) The radial in-fall velocity of matter increases with the time of the co-moving observer in any unstoppable collapse. The condition $\Delta = 0$ is then reached for some part of matter in the spacetime of (4) when the temporal metric function $B(t)$ satisfies (32).

To continue with the analysis of the light trapping surface, we note that conditions (40) - (42) possess an immediate interpretation. Noticing that the "physical" radial distance is $\gamma(y')B$, these conditions imply that the physical distance covered per unit co-moving time with velocity $V_r$ is less than, equal to or greater than the distance measured by $y$. That is, if we use $y$ as the radial coordinate (with $y(r=0) = y_c \neq 0$), the conditions (40) - (42) have the above interpretation. In particular, at the outermost Light Trapping Surface, we expect the in-fall velocity $V_r = 1$ and, hence, the condition (41) implies that the "coordinate" distance $y$ equal the light-travel time from this surface to the singularity at $B(t_s) = 0$ at this surface. We note that this is a very natural interpretation of the condition (41). (See, also, (34).)

Then, matter shells, in their rest frame, cross the Light Trapping Surface in successions when the shell-labelling radial coordinate, $y$, corresponds to the light-crossing time between the LTS and the singularity at $B(t_s) = 0$. Therefore, matter within the region $y \leq \gamma(y')B$ gets trapped inside the outmost light-trapping surface when the collapse advances to satisfy the condition (32) on the temporal metric function $B$.

**Initial conditions**

We note that the initial conditions for the spacetime of (4) consist of conditions at the "initial co-moving time" and not for large radial distances. Primarily, the 'initial' temporal functions are to be chosen on the basis of the choice of the radial function $y(r)$. If the 'initial' radial density distribution is such as to result to immediate heat generation then the temporal metric function $B$ is not initially constant. So, is the case with 'initial' value of shear in the spacetime. Therefore, the nature of 'initial' temporal and spatial metric functions is to be decided on the basis of the 'astrophysical' nature of the problem under consideration.

When we consider the gravitational collapse of matter from some initial density distribution corresponding to a "small" over-density, the initial heat or radiation flow in the spacetime could be very small. Then, in the absence of any ionizing radiation, matter in the spacetime could be expected to be "dusty" ie, approximately pressureless with negligible heat flow at initial co-moving time.

Moreover, matter may, at initial co-moving time, be assumed to be non-relativistic throughout the spacetime and, hence, we have

$$V_r \equiv V_{ini} << 1 \qquad \text{(early time)} \qquad (43)$$

We may also expect negligible radiation in the spacetime at initial co-moving time, indicating that $B \approx$ constant initially. Then, initially,

$$\Delta \approx 1 - \gamma^2 \left(\frac{y'}{y}\right)^2 B^2 V_{ini}^2 \approx 1 \qquad (44)$$

However, heat flow eventually grows. In this connection, we emphasize that a star forms at some suitable co-moving time and emits radiation that flows through the medium surrounding it. For large distances from the star, the radiation flux is weak and, hence, non-ionizing. It is then understandable that the spacetime of (4) does not admit a collapsing dust solution without heat flux since radiation must flow in the spacetime at large $r$.

It may be noted that the large $r$ condition of matter is to be inferred on the basis of only the astrophysical expectation that matter will be non-relativistic and dusty far away from a source of intense radiation. This is the sort of situation that is commonly observed with accreting objects.

We emphasize here that this expectation is, automatically, borne out by the spacetime of (4) on the basis of the radial dependence of various physical quantities. The condition of matter for large $r$ is, however, not essential to the physics of the gravitational collapse beyond specifying the properties of matter in the far-away zone. Then, close

to the central object the equation of state of matter is expected to be different. This is precisely the situation with the spacetime of (4).

Further, describing the fluid by its local thermodynamical properties [11], the energy conservation principle implies the first law of thermodynamics in the form

$$I_{,a}U^a \;=\; -\mathcal{C} \,-\, p\left(\frac{1}{n}\right)_{,a}U^a \qquad (45)$$

where we have used $\rho \,=\, n\,(1+I)$ with $I$ as the specific internal energy, $\mathcal{C}(T,n)$ as the rate of decrease of internal energy per unit amount of matter and $n$ as the number density of particles of matter. The function $\mathcal{C}$ is related to the heat flux $q$.

Note also that, in general, we can write

$$V_r \;=\; \frac{yY}{2}\,\left[\,(2\eta)\,3\,\sigma \,-\, \gamma^2 B^2 (yy')\,q\,\right] \qquad (46)$$

Some of the standard analysis [12] of the accretion process can then be followed for the spacetime of (4) by considering appropriate initial conditions.

However, our purpose here is limited only to showing that a black hole forms in the gravitational collapse only as an infinite red-shift surface that is not a null hyper-surface. Therefore, we do not consider here the details of this analysis which will, however, be the subject of our future works. But, we have presented here all the details which are essential to consider the problem of spherical accretion in its totality.

## IV. CONCLUDING REMARKS

In [9] we showed that naked singularities do not form in spherically symmetric, spatially homothetic spacetimes for non-singular, initial data for matter fields. In [10], we considered the shear-free gravitational collapse and its implications for the Cosmic Censorship Hypothesis. In [7], we showed that the Cosmic Censorship Hypothesis is equivalent to the statement that gravity has no length-scale for matter properties. Further to all of the above, we showed in this paper that the spatially homothetic, spherically symmetric spacetime of (4) leads to a very general description of the formation of a spherical object in General Relativity. In particular, we showed that the spacetime of (4) admits a black hole only as an infinite red-shift surface and not as a null hyper-surface.

Further, considering that the spacetime of (4) admits any equation of state for matter in the spacetime and that it is the only spherically symmetric spacetime satisfying the spatial scale-invariance of gravity, we emphasize that black holes obtained in spherical gravitational collapse in Nature must necessarily be of the type considered here.

The description of spherical gravitational collapse, presented in this paper, is also applicable to the problem of spherical accretion onto a central gravitating object. We have displayed all the equations necessary for any detailed analysis of this important astrophysical problem that will be dealt with in future subsequent works.

As a final remark, we note that many features of spherical gravitational collapse and accretion processes considered here will be obtainable for general spatially homothetic spacetime [5]. In particular, the initial conditions will be in terms of the temporal metric functions and the spatial density distribution. The light trapping property of gravity in terms of the formation of the light trapping surface will also correspond to some "instant" of the co-moving time. A general spatially homothetic spacetime will not admit a null hyper-surface but a black hole can form in it only as an infinite red-shift surface.

The nature of the black hole that we obtain for spatially homothetic spacetimes is very similar to that obtainable [13] in the newtonian theory of gravity except that our considerations here are fully general relativistic. The black hole of (4) is only an infinite red-shift surface and not a null hyper-surface. In general, the black hole of the general spatially homothetic spacetime [5] can be expected to be only an infinite red-shift surface. It is then to be noted that these results will have important implications for observational astrophysics and for other considerations in General Relativity [14] - [17] that have been based on the conception of a black hole as a null hyper-surface.


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